Cambridge Assessment International Education

# Cambridge International AS & A Level

**MATHEMATICS** **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2021**

MARK SCHEME

Maximum Mark: 75

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.



© UCLES 2021



https://xtremepape.rs/

https://xtremepape.rs/



## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

| |
|---|
| GENERIC MARKING PRINCIPLE 1:<br>Marks must be awarded in line with:<br>• the specific content of the mark scheme or the generic level descriptors for the question<br>• the specific skills defined in the mark scheme or in the generic level descriptors for the question<br>• the standard of response required by a candidate as exemplified by the standardisation scripts. |
| GENERIC MARKING PRINCIPLE 2:<br>Marks awarded are always **whole marks** (not half marks, or other fractions). |
| GENERIC MARKING PRINCIPLE 3:<br>Marks must be awarded **positively**:<br>• marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate<br>• marks are awarded when candidates clearly demonstrate what they know and can do<br>• marks are not deducted for errors<br>• marks are not deducted for omissions<br>• answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous. |
| GENERIC MARKING PRINCIPLE 4:<br>Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors. |
| GENERIC MARKING PRINCIPLE 5:<br>Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen). |
| GENERIC MARKING PRINCIPLE 6:<br>Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind. |

https://xtremepape.rs/



| Mathematics Specific Marking Principles | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
| 4 | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw). |
| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

© UCLES 2021 

https://xtremepape.rs/



## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

© UCLES 2021

https://xtremepape.rs/



**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

https://xtremepape.rs/



| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | $(4x-3)^2$ or $(4x+(-3))^2$ or $a=-3$ | B1 | $k(4x-3)^2$ where $k \neq 1$ scores B0 but mark final answer, allow recovery. |
| | $+1$ or $b=1$ | B1 | |
| | | **2** | |
| 1(b) | [For one root] $k=1$ or '*their b*' | B1 FT | Either by inspection or solving or from $24^2-4\times16\times(10-k)=0$ WWW |
| | $[\text{Root or } x=]\dfrac{3}{4}$ or 0.75 | B1 | **SC B2** for correct final answer WWW. |
| | | **2** | |

https://xtremepape.rs/

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | Translation $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ | B1 | Allow shift and allow by 1 in $x$-direction or [parallel to/on/in/along/against] the $x$-axis or horizontally.<br>'Translation by 1 to the right' only, scores B0 |
| | Stretch | B1 | Stretch. **SC B2** for amplitude doubled. |
| | Factor 2 in $y$-direction | B1 | With/by **factor** 2 in $y$-direction or [parallel to/on/in/along/against] the $y$-axis or vertically or with $x$ axis invariant<br>'With/by factor 2 upwards' only, scores B0.<br>Accept SF as an abbreviation for scale factor. |
| | | **3** | **Note:** Transformations can be in either order |
| 2(b) | [$-\sin 6x$][ $+ 15x$] or [$\sin(-6x)$][ $+ 15x$] OE | B1 B1 | Accept an unsimplified version. ISW.<br>B1 for each correct component – square brackets indicate each required component. |
| | | | If B0, **SC B1** for either $\sin(-2x) + 5x$ or $-\sin(2x) + 5x$ or $\sin 6x - 15x$ or $\sin\left(-\frac{2}{3}x\right)+\frac{5}{3}x$ |
| | | **2** | |

https://xtremepape.rs/



| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | 1.2679 | B1 | AWRT. ISW if correct answer seen. $3-\sqrt{3}$ scores B0 |
| | | **1** | |
| 3(b) | 1.7321 | B1 | AWRT. ISW if correct answer seen. |
| | | **1** | |
| 3(c) | Sight of 2 or 2.0000 or two in reference to the gradient | *B1 | |
| | This is because the gradient at $E$ is the limit of the gradients of the chords as the $x$-value tends to 3 or $\delta x$ tends to 0. | DB1 | Allow it gets nearer/approaches/tends/almost/approximately 2 |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | [Coefficient of $x$ or $p$ =] 480 | B1 | SOI. Allow $480x$ even in an expansion. |
| | $\left[\text{Term in } \frac{1}{x} \text{ or } q=\right][10\times](2x)^3\left(\frac{k}{x^2}\right)^2$ | M1 | Appropriate term identified and selected. |
| | $[10\times2^3k^2=]\ 80k^2$ | A1 | Allow $\frac{80k^2}{x}$ |
| | $p=6q$ used $(480=6\times80k^2$ or $80=80k^2)$ | M1 | Correct link used for *their* coefficient of $x$ and $\frac{1}{x}$ ($p$ and $q$) with no $x$'s. |
| | $[k^2=1\Rightarrow]\ k=\pm1$ | A1 | A0 if a range of values given. Do not allow $\pm\sqrt{1}$. |
| | | **5** | |

https://xtremepape.rs/



| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $\text{ff}(x) = 2(2x^2 + 3)^2 + 3$ | M1 | Condone = 0. |
| | $8x^4 + 24x^2 + 21$ | A1 | ISW if correct answer seen. Condone = 0. |
| | | **2** | |
| 5(b) | $8x^4 + 24x^2 + 21 = 34x^2 + 19 \Rightarrow 8x^4 + 24x^2 - 34x^2 + 21 - 19$ [= 0] | M1 | Equating $34x^3 + 19$ to *their* 3-term ff($x$) and collect all terms on one side condone ± sign errors. |
| | $8x^4 - 10x^2 + 2$[= 0] | A1 | |
| | [2]$(x^2 - 1)(4x^2 - 1)$ | M1 | Attempt to solve 3-term quartic or 3-term quadratic by factorisation, formula or completing the square or factor theorem. |
| | $\left[x^2 = 1 \text{ or } \frac{1}{4} \text{ leading to}\right] x = 1 \text{ or } x = \frac{1}{2}$ | A1 | If factorising, factors must expand to give $8x^4$ or $4x^4$ 4 or *their* $ax^4$ otherwise M0A0 due to calculator use. Condone $\pm 1$, $\pm\frac{1}{2}$ but not $\sqrt{\frac{1}{4}}$ or $\sqrt{1}$. |
| | | **4** | |

https://xtremepape.rs/



| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6 | Gradient AB = $\frac{1}{2}$ | B1 | SOI |
| | Lines meet when $-2x+4=\frac{1}{2}(x-8)+3$<br>Solving as far as $x =$ | *M1 | Equating given perpendicular bisector with the line through (8, 3) using *their* gradient of *AB* (but not -2) and solving.<br>Expect $x = 2$, $y = 0$. |
| | Using mid-point to get as far as $p =$ or $q =$ | DM1 | Expect $\frac{8+p}{2}=2$ or $\frac{3+q}{2}=0$ |
| | $p = -4$, $q = -3$ | A1 | Allow coordinates of *B* are $(-4, -3)$. |
| | **Alternative method for Question 6** | | |
| | Gradient AB = $\frac{1}{2}$ | B1 | SOI |
| | $\frac{q-3}{p-8}=\frac{1}{2}$ [leading to $2q = p-2$],<br>$\frac{q+3}{2}=-2\left(\frac{8+p}{2}\right)+4$ [leading to $q=-11-2p$] | *M1 | Equating gradient of *AB* with *their* gradient of *AB* (but not -2) and using mid-point in equation of perpendicular bisector. |
| | Solving simultaneously *their* 2 linear equations | DM1 | Equating and solving 2 correct equations as far as $p =$ or $q =$ . |
| | $p = -4$, $q = -3$ | A1 | Allow coordinates of *B* are $(-4, -3)$. |

https://xtremepape.rs/



| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6 | **Alternative method for Question 6** | | |
| | Gradient AB = $\frac{1}{2}$ | B1 | |
| | $\frac{q-3}{p-8}=\frac{1}{2}$ $\left[\text{leading to } p=2q+2\right]$,<br>$y-\frac{q+3}{2}=-2\left(x-\left(q+5\right)\right)$ $\left[\text{leading to } y=-2x+\frac{5q+23}{2}\right]$ | *M1 | Equating gradient of *AB* with *their* gradient of *AB* (but not -2) and using mid-point in equation of perpendicular bisector. |
| | *their* $\frac{5q+23}{2}=4 \Rightarrow q=$ | DM1 | Equating and solving as far as $q$ or $p$ = |
| | $p=-4,\ q=-3$ | A1 | Allow coordinates of $B$ are $(-4, -3)$. |
| | | **4** | |

https://xtremepape.rs/



| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $(5-1)^2 + (11-5)^2 = 52$ or $\dfrac{11-5}{5-1}$ | M1 | For substituting (1,5) into circle equation or showing gradient $= \dfrac{3}{2}$. |
| | For both circle equation and gradient, and proving line is perpendicular and stating that *A* lies on the circle | A1 | Clear reasoning. |
| | **Alternative method for Question 7(a)** | | |
| | $(x-5)^2 + (y-11)^2 = 52$ and $y-5 = -\dfrac{2}{3}(x-1)$ | M1 | Both equations seen and attempt to solve. May see $y = -\dfrac{2}{3}x + \dfrac{17}{3}$ |
| | Solving simultaneously to obtain $(y-5)^2 = 0$ or $(x-1)^2 = 0 \Rightarrow 1$ root or tangent or discriminant $= 0 \Rightarrow 1$ root or tangent | A1 | Clear reasoning. |
| | **Alternative method for Question 7(a)** | | |
| | $\dfrac{dy}{dx} = \dfrac{10-2x}{2y-22} = \dfrac{10-2}{10-22}$ | M1 | Attempting implicit differentiation of circle equation and substitute $x = 1$ and $y = 5$. |
| | Showing gradient of circle at A is $-\dfrac{2}{3}$ | A1 | Clear reasoning. |
| | | **2** | |
| 7(b) | Centre is $(-3, -1)$ | B1 B1 | B1 for each correct co-ordinate. |
| | Equation is $(x+3)^2 + (y+1)^2 = 52$ | B1 FT | FT *their* centre, but not if either (1, 5) or (5, 11). Do not accept $\sqrt{52^2}$. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | $\left(a+b=2\times\frac{3}{2}a\right)\Rightarrow b=2a$ | B1 | SOI |
| | $18^2 = a(b+3)$ OE or 2 correct statements about $r$ from the GP,<br>e.g. $r=\frac{18}{a}$ and $b + 3 = 18r$ or $r^2=\frac{b+3}{a}$ | B1 | SOI |
| | $324 = a(2a + 3) \Rightarrow 2a^2 + 3a - 324[= 0]$<br>or<br>$b^2 + 3b - 648[= 0]$<br>or<br>$6r^2 - r - 12[= 0]$<br>or<br>$4d^2 + 3d - 162[= 0]$ | M1 | Using the correct connection between AP and GP to form a 3-term quadratic with all terms on one side. |
| | $(a - 12)(2a + 27)[= 0]$<br>or<br>$(b-24)(b+27)[=0]$<br>or<br>$(2r-3)(3r+4)[=0]$<br>or<br>$(d-6)(4d+27)[=0]$ | M1 | Solving *their* 3-term quadratic by factorisation, formula or completing the square to obtain answers for *a, b, r* or *d*. |
| | $a = 12$, $b = 24$ | A1 | WWW. Condone extra 'solution' $a=-13.5, b=-27$ only. |
| | | **5** | |



https://xtremepape.rs/

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(b) | Common difference $d = 6$ | B1 FT | SOI. FT *their* $\frac{a}{2}$ |
| | $S_{20} = \frac{20}{2}(2\times12+19\times6)$ | M1 | Using correct sum formula with *their a, their* calculated $d$ and 20. |
| | 1380 | A1 | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9 | Curve intersects $y = 1$ at (3, 1) | B1 | **Throughout Question 9: 1 < *their* 3 < 5**<br>Sight of $x = 3$ |
| | Volume $= [\pi]\int(x-2)[\mathrm{d}x]$ | M1 | M1 for showing the intention to integrate $(x-2)$.<br>Condone missing $\pi$ or using $2\pi$. |
| | $[\pi]\left[\frac{1}{2}x^2-2x\right]$ or $[\pi]\left[\frac{1}{2}(x-2)^2\right]$ | A1 | Correct integral. Condone missing $\pi$ or using $2\pi$. |
| | $=[\pi]\left[\left(\frac{5^2}{2}-2\times5\right)-\left(\frac{their\,3^2}{2}-2\times their\,3\right)\right]$<br>$=[\pi]\left[\frac{5}{2}+\frac{3}{2}\right]$ as a minimum requirement for *their* values | M1 | Correct use of '*their* 3' and 5 in an integrated expression.<br>Condone missing $\pi$ or using $2\pi$. Condone +c.<br>Can be obtained by integrating and substituting between 5 and 2 and then 3 and 2 then subtracting. |
| | Volume of cylinder $= \pi\times1^2\times(5-their\,3)[=2\pi]$ | B1 FT | Or by integrating 1 to obtain $x$ (condone $y$ if 5 and *their* 3 used). |
| | [Volume of solid $= 4\pi-2\pi =$]$2\pi$ or 6.28 | A1 | AWRT |



https://xtremepape.rs/

https://xtremepape.rs/



| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9 | **Alternative method for Question 9** | | |
| | Curve intersects $y = 1$ at (3, 1) | B1 | Sight of $x = 3$ |
| | Volume of solid = $\pi\int(x-2)-1[\mathrm{d}x]$ | M1 B1 | M1 for showing the intention to integrate $(x-2)$<br>B1 for correct integration of –1.<br>Condone missing $\pi$ or $2\pi$ for M1 but not for B1. |
| | $[\pi]\left[\frac{1}{2}x^2-3x\right]$ or $[\pi]\left[\frac{1}{2}(x-3)^2\right]$ | A1 | Correct integral, allow as two integrals.<br>Condone missing $\pi$ or using $2\pi$. |
| | $=[\pi]\left[\left(\frac{5^2}{2}-3\times5\right)-\left(\frac{their\,3^2}{2}-3\times their\,3\right)\right]$ | M1 | Correct use of '*their* 3' and 5 in an integrated expression.<br>Condone missing $\pi$ or using $2\pi$. Condone +c.<br>Can be obtained by integrating and substituting between 5 and 2 and then 3 and 2 then subtracting. |
| | [Volume of solid = $4\pi-2\pi=$]$2\pi$ or 6.28 | A1 | AWRT |
| | | **6** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(a) | $\frac{1+\sin x}{1-\sin x}-\frac{1-\sin x}{1+\sin x}\equiv\frac{(1+\sin x)^2-(1-\sin x)^2}{(1-\sin x)(1+\sin x)}$ | *M1 | For using a common denominator of $(1-\sin x)(1+\sin x)$ and reasonable attempt at the numerator(s). |
| | $\equiv\frac{1+2\sin x+\sin^2 x-(1-2\sin x+\sin^2 x)}{(1-\sin x)(1+\sin x)}$ | DM1 | For multiplying out the numerators correctly. Condone sign errors for this mark. |
| | $\equiv\frac{4\sin x}{1-\sin^2 x}\equiv\frac{4\sin x}{\cos^2 x}$ | DM1 | For simplifying denominator to $\cos^2 x$. |
| | $\equiv\frac{4\sin x}{\cos x\cos x}\equiv\frac{4\tan x}{\cos x}$ | A1 | AG.<br>Do not award A1 if undefined notation such as s, c, t or missing $x$'s used throughout or brackets are missing. |
| | **Alternative method for Question 10(a)** | | |
| | $\frac{4\tan x}{\cos x}\equiv\frac{4\sin x}{\cos^2 x}\equiv\frac{4\sin x}{1-\sin^2 x}$ | *M1 | Using $\tan x=\frac{\sin x}{\cos x}$ and $\cos^2 x= 1-\sin^2 x$ |
| | $\equiv\frac{-2}{1+\sin x}+\frac{2}{1-\sin x}$ | DM1 | Separating into partial fractions. |
| | $\equiv 1+\frac{-2}{1+\sin x}+\frac{2}{1-\sin x}-1$ | DM1 | Use of 1-1 or similar |
| | $\equiv-\frac{1-\sin x}{1+\sin x}+\frac{1+\sin x}{1-\sin x}$ | A1 | |
| | | **4** | |



https://xtremepape.rs/

https://xtremepape.rs/



| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(b) | $\cos x = \frac{1}{2}$ | *B1 | OE. WWW. |
| | $x = \frac{\pi}{3}$ | DB1 | Or AWRT 1.05 |
| | $x = 0$ from $\tan x = 0$ or $\sin x = 0$ | B1 | WWW. Condone extra solutions outside the domain 0 to $\frac{\pi}{2}$ but B0 if any inside. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 11(a) | At stationary point $\frac{dy}{dx} = 0$ so $6(3\times 2-5)^3 - k\times 2^2 \ = 0$ | M1 | Setting given $\frac{dy}{dx} = 0$ and substituting $x = 2$ into it. |
| | $[k=]\frac{3}{2}$ | A1 | OE |
| | | **2** | |
| 11(b) | $[y=]\frac{6}{4\times 3}(3x-5)^4 - \frac{1}{3}kx^3\ [+c]$. | *M1<br>A1FT | Integrating (increase of power by 1 in at least one term) given $\frac{dy}{dx}$. Expect $\frac{1}{2}(3x-5)^4 - \frac{1}{2}x^3$.<br>FT *their* non zero $k$. |
| | $-\frac{7}{2} = \frac{1}{2}(3\times 2-5)^4 - \frac{1}{3}\times\frac{3}{2}\times 2^3 + c$ [leading to $-3.5+c=-3.5$] | DM1 | Using (2,-3.5) in an integrated expression. + $c$ needed.<br>Substitution needs to be seen, simply stating $c$ = 0 is DM0. |
| | $y = \frac{1}{2}(3x-5)^4 - \frac{1}{2}x^3$ | A1 | $y$ = or f($x$)= must be seen somewhere in solution. |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 11(b) | **Alternative method for Question 11(b)** | | |
| | $[y=]\dfrac{81}{2}x^4-\dfrac{541}{2}x^3+675x^2-750x(+c)$ or $-270\,x^3-k\dfrac{x^3}{3}$ | *M1<br>A1 FT | From $\dfrac{dy}{dx}=162x^3-810x^2-kx^2-1350x-750$. FT *their k* |
| | $-\dfrac{7}{2}=\dfrac{81}{2}\times2^4-\dfrac{541}{2}\times2^3+675\times2^2-750\times2+c$ | DM1 | Using (2, -3.5) in an integrated expression. + $c$ needed |
| | $y=\dfrac{81}{2}x^4-\dfrac{541}{2}x^3+675x^2-750x+\dfrac{625}{2}$ | A1 | $y$ = or f($x$)= must be seen somewhere in solution. |
| | | **4** | |
| 11(c) | $[3\times]\left[18(3x-5)^2\right][-2kx]$ | B2,1,0 FT | FT *their k.*<br>Square brackets indicate each required component.<br>B2 for fully correct, B1 for one error or one missing component, B0 for 2 or more errors. |
| | **Alternative method for Question 11(c)** | | |
| | $486x^2-1623x+1350$ or $-1620x-2kx$ | B2,1,0 FT | FT *their k.*<br>B2 for fully correct, B1 for one error, B0 for 2 or more errors. |
| | | **2** | |
| 11(d) | [At $x=2$] $\left[\dfrac{d^2y}{dx^2}=\right]54(3\times2-5)^2-4k$ or 48 | M1 | OE. Substituting $x$ = 2 into *their* second differential or other valid method. |
| | [> 0] Minimum | A1 | WWW |
| | | **2** | |

https://xtremepape.rs/



| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 12(a) | [By symmetry] [6 × $P\hat{A}Q$ = 2π], [ $P\hat{A}Q$ =] 2π÷6, | M1 | |
| | Explaining that there are six sectors around the diagram that make up a complete circle. | A1 | AG |
| | **Alternative method for Question 12(a)** | | |
| | Using area or circumference of circle centre $A$ ÷ 6 | M1 | $\frac{400\pi}{6}$ or $\frac{40\pi}{6}$ |
| | Justification for dividing by 6 followed by comparison with the sector area or arc length. | A1 | AG |
| | **Alternative method for Question 12(a)** | | |
| | Explain why $\Delta PAQ$ is an equilateral triangle | M1 | Assumption of this scores M0 |
| | Using $\Delta PAQ$ is an equilateral triangle $\therefore$ $P\hat{A}Q = \frac{\pi}{3}$ | A1 | AG |
| | **Alternative method for Question 12(a)** | | |
| | Using the internal angle of a regular hexagon = $\frac{2\pi}{3}$<br>Or $F\hat{A}O + O\hat{A}B = \frac{2\pi}{3}$ , equilateral triangles | M1 | |
| | $P\hat{A}Q = 2\pi - \left(\frac{\pi}{2} + \frac{2\pi}{3} + \frac{\pi}{2}\right) = \frac{\pi}{3}$ | A1 | AG |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 12(a) | **Alternative method for Question 12(a)** | | |
| | $Sin\theta = \frac{20}{40}$, with $\theta$ clearly identified | M1 | |
| | $\theta = \frac{\pi}{6}, 2\theta = \frac{\pi}{3} = F\hat{A}O$ and by similar triangles $= P\hat{A}Q$ | A1 | AG |
| | | **2** | |
| 12(b) | Each straight section of rope has length 40 cm | B1 | SOI |
| | Each curved section round each pipe has length $r\theta = 20\times\frac{\pi}{3}$ | *M1 | Use of $r\theta$ with $r = 20$ and $\theta$ in radians |
| | Total length $= 6\times\left(\left(their\,40\right) + k\pi\right)$ | DM1 | $6\times$(*their* straight section + *their* curved section).<br>*Their* curved section must be from acceptable use of $r\theta$ – this could now be numeric. |
| | $240 + 40\pi$ or 366 (AWRT) (cm) | A1 | Or directly: ($6\times$ diameter) + circumference |
| | | **4** | |



https://xtremepape.rs/

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 12(c) | [Triangle area =] $\frac{1}{2}\times 40\times 40\times \sin\left(\frac{\pi}{3}\right)$ or $\frac{1}{2}\times 40\times 20\sqrt{3}$ or $400\sqrt{3}$ or 693(AWRT) | B1 | |
| | [Total area of hexagon = 6 × $400\sqrt{3}$ =] $2400\sqrt{3}$ | B1 | Condone $4800\frac{\sqrt{3}}{2}$ |
| | **Alternative method for Question 12(c)** | | |
| | [Trapezium area =] $\frac{1}{2}\times(40+80)\times 40\sin\left(\frac{\pi}{3}\right)$ or $1200\sqrt{3}$ or 2080 (AWRT) | B1 | |
| | [Total area of hexagon = 2 × $1200\sqrt{3}$ =] $2400\sqrt{3}$ | B1 | Condone $4800\frac{\sqrt{3}}{2}$ |
| | **Alternative method for Question 12(c)** | | |
| | Area of triangle *ABC* = $400\sqrt{3}$ or 693 (AWRT) or<br>4 × Area of half of triangle *ABC* = $4\times 200\sqrt{3}$ or 1390 (AWRT)<br>or Area of rectangle *ABDE* = $1600\sqrt{3}$ or 2770 (AWRT) | B1 | |
| | [Total area of hexagon = $2\times 400\sqrt{3}+1600\sqrt{3}$ =] $2400\sqrt{3}$<br>Or [= $4\times 200\sqrt{3}+1600$ =] $2400\sqrt{3}$ | B1 | Condone $4800\frac{\sqrt{3}}{2}$ |
| | | | If B0B0, **SC B1** can be scored for sight of 4160 (AWRT) as final answer. |
| | | **2** | |

https://xtremepape.rs/



| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 12(d) | Each rectangle area = $40 \times 20$ (= 800) | B1 | SOI, e.g. by sight of 4800 |
| | Each sector area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 20^2 \times \frac{\pi}{3}\left[=\frac{200\pi}{3}\right]$ | B1 | SOI. |
| | Total area = $2400\sqrt{3} + 4800 + 400\pi$ or 10 200 (cm$^2$) (AWRT) | B1 | Or directly: part **(c)** + 6800 + area circle radius 20. |
| | | **3** | |